



QUANTUM COMPUTING IN FLEET MANAGEMENT

Abstract

The vast and rapid business expansion in FMCG, courier services, and transportation sectors and the fast pace of delivery commitments have accelerated the need for an efficient route selection. The complexity of the problem increases with more constraints like - geolocation data of factories, distribution centres, delivery destinations, maximum capacity constraints of a distribution centre, demand at delivery destinations, and the transportation vehicle capacity - coming into the picture. The cascading effect it has on the industry at large is immense. At present we look at quantum computers for a solution to address and effectively overcome the conundrum.

Introduction

The problem of finding the optimal route for a vehicle to deliver and collect products at/from multiple destinations is called the Vehicle Routing Problem (VRP). VRP aims to find a cost-effective route for various vehicles to serve several destinations. The issue is more enduring for the organizations relying upon state-of-art classical computers where the challenge increases exponentially with scaling complexities [1]. Finding the optimum route belongs to a class of 'Combinatorial Optimization Problems'. These problems involve the selection of an optimal solution from a stock of various combinations. For this selection, the computer has to evaluate all the available possibilities. This type of problem is categorised under the Nondeterministic Polynomial-time Hard (NP-Hard) problems [2]. NP-Hard problems are those problems that a classical computer cannot solve in a practical time.

Fleet Management Market

The Global Fleet Management market, estimated at USD 19.9 Billion in 2020, is projected to reach USD 34 Billion by 2025. The Compounded Annual Growth Rate (CAGR) for Fleet Management is 11.3% as per MarketsandMarkets. The market growth is stimulated by the adoption of cloud and IoT based services. Operational efficiency, cost efficiency, and time management have also aided the technological advancement in fleet management. The North American region has the highest market share in fleet management due to the early adoption of advanced solutions [3].

Challenges in Optimizing Vehicle Routing

Optimizing Vehicle Routing by mathematical calculations involves dealing with various combinations of conditions and constraints. These make the simulation of VRP's Mathematical model complex. The

following are the examples of some of the complications in the VRP [4]:

- Vehicle Routing Problem with Profits (VRPP), where visiting only the destinations that maximize the profit is required.
- Vehicle Routing Problem with Pickup and Delivery (VRPPD), where each vehicle picks up items and drops them to delivery points.
- Capacitated Vehicle Routing Problem: CVRP or CVRPTW.
- Vehicle Routing Problem with Multiple Trips (VRPMT).
- Inventory Routing Problem (IRP).
- Multi-Depot Vehicle Routing Problem (MDVRP).

Similarly, every variation involves additional constraints like the vehicle capacity, priority time windows, multiple depots, destination locations, inventory constraints, multiple trips, etc. These variations in a VRP give several choices to deal with to make a cost-effective decision. This process requires accessing all the available options by calculating and selecting that combination of routes that minimize the cost of routing as well as satisfy all the conditions and constraints such as Priority orders, Pickup and Delivery, Time windows, Multiple trips, etc. A typical workorder dashboard is shown in Figure 01.

A classical computer uses 1's and 0's for computation. Solving complex mathematical models through this approach becomes challenging because the complexity increases multiple folds with even a small increase in data points. So, the time taken for a classical computer to find an optimal solution grows exponentially with increasing data points, and quickly reaches a state where it cannot be solved in real-time.

A quantum computer follows a different approach: instead of 1's and 0's, it uses

concepts of quantum mechanics like superposition and entanglement to calculate optimal solution faster.

The Classical Approach

The overburdening of classical computers caused by the constraints mentioned above drives the adoption of Heuristic Methods for approximating optimum routes and making the problem classically computable [5]. In the Heuristic approach to problem-solving, we use a practical method or various shortcuts to produce solutions that may not be optimal but are sufficient, given a limited timeframe. This trade-off of time with the accuracy of Heuristic approaches can accelerate the process but the solution thus concluded is not precise, amounting to losses that the enterprise may incur.

The solution to the VRP as a combinatorial optimization problem involves two steps:

1. Encoding the problem statement into a mathematical model.
2. Getting a solution by feeding input data to the mathematical model.

The mathematical modelling starts with a basic TSP (Travelling Salesman Problem) model:

$$C_1(x) = \sum_i^n \sum_j^n w_{ij} x_{ij}$$

Where,

- w_{ij} is the cost of traveling from destination i to j .
- x_{ij} is the binary decision variable

Constraints are specific to every problem statement.

The solution to the above mathematical model involves accessing every possible combination and selecting the minimum cost combination. For a basic TSP problem without many conditions or constraints, the number of possible combinations

increases by (n-1)!

Here is how the number of computations increases:

- 5 nodes = 24 computations
- 10 nodes = 362880 computations
- 15 nodes = 8.72E+10 computation

In this example, by just increasing the data points from 10 to 15, the number of computations increases. For a computer using the 0s and 1s approach, this increase takes a longer time to find a minimum solution disrupting the business timeline. Classical computations for VRP become expensive when the number of destinations or delivery points scales to hundreds. The introduction of more conditions and constraints in the problem reduces this window of classical computation capability even further.

This issue of classical computers getting obsolete for a complex problem leaves a huge void for enterprises and pushes them to look for better alternatives to these issues

The Quantum Solution

Quantum algorithms have proven to speed up the computations drastically and give the results in polynomial-time [6]. Before using these algorithms, we need to map our mathematical model to a Hamiltonian. The Hamiltonian is a mathematical function involving coordinates and momenta (and possibly time), which typically represents the energy of a dynamical system. Ising-Hamiltonian is the most preferred Hamiltonian for mathematical models with binary variables as it correlates with the Quadratic Unconstrained Binary Optimization (QUBO)

models [7] [8].

After mapping the mathematical model to a Hamiltonian, we can use any of the following two algorithms - Variational Quantum Eigensolver (VQE) or Quantum Approximate Optimization Algorithm (QAOA) using quantum computers [9].

VQE is an algorithm based on the Rayleigh-Ritz Variational Principle. It is a hybrid algorithm constituting quantum and classical parts. The quantum part sets up the quantum wave function measuring the Hamiltonian expectation value on this wave function. The result thus obtained is then passed to the classical part. The classical part comes up with new angles of the unitary rotations so that further iterations can yield a lower Hamiltonian Expectation value. This process is performed iteratively till the ground state energy is achieved. This offers a solution to the minimization problem [10].

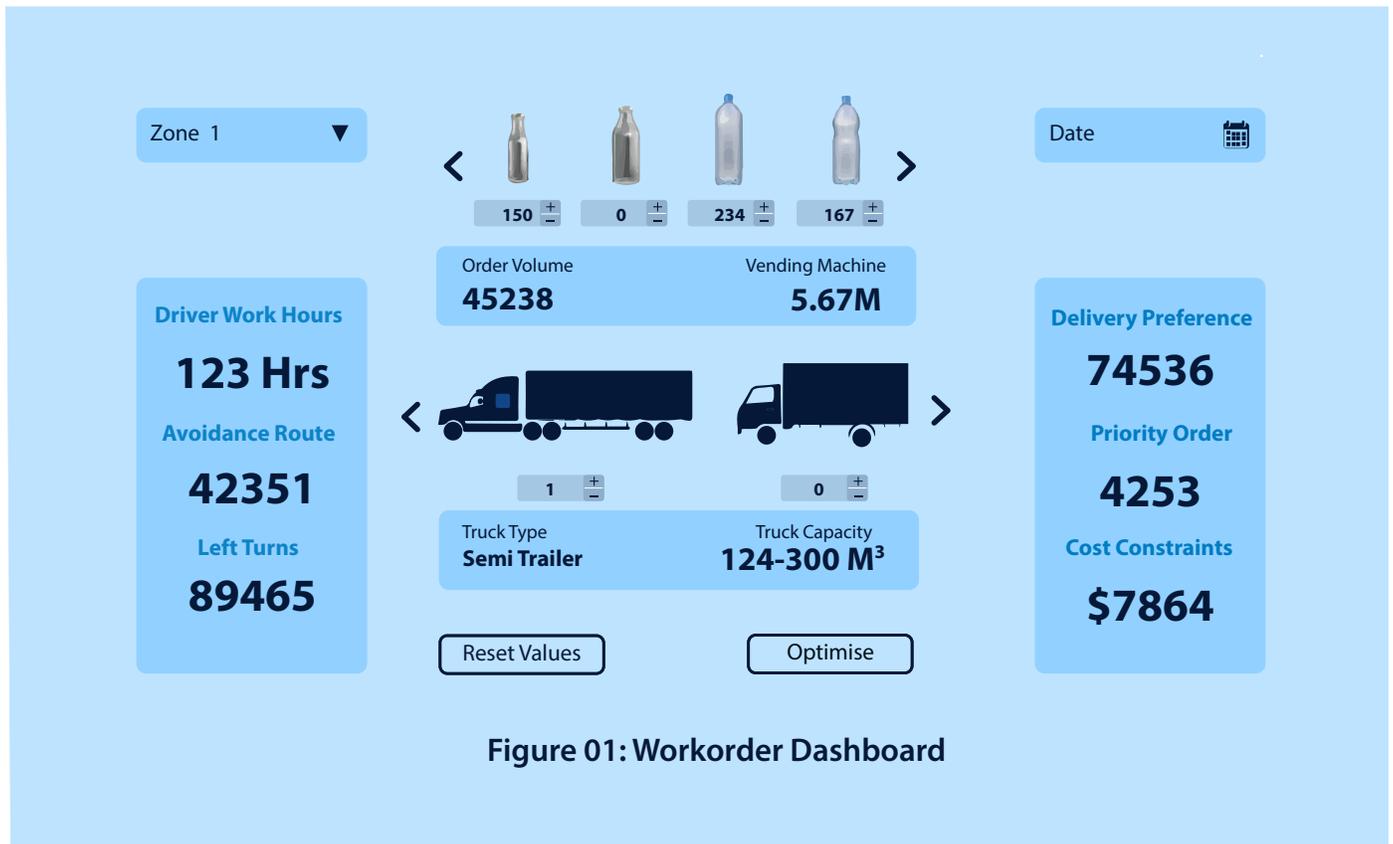


Figure 01: Workorder Dashboard

QAOA algorithm is based on the principle of "Quantum Annealing". It is said to exhibit Quantum Supremacy, i.e. it can solve problems that no other classical computer can solve. The computational power of a QAOA algorithm running on 420 qubits and 500 constraints equates to that of a classical supercomputer running for one century [11].

Using these algorithms, a quantum computer made of n Qubits can perform up to 2^n calculations in parallel, whereas a classical computer made of n Bits can

perform only n operations. Hence, with an exponential increase in the complexity of the problem, the computing power of a quantum computer is also boosted exponentially by the Qubits. The approach of superposition and entanglement enables the quantum computer to simulate the exponentially growing conditions and constraints efficiently by mapping them to a Wave Function Equation. This helps quantum computers in finding the minimum energy solution of the wave function, and in turn, the minimum solution of the VRP. This is

how a quantum computer converges to the solution in a practical time where a classical computer can take years.

Figure 02 shows a comparison of the computation time for a simple VRP running on a classical versus a quantum computer. The graph is extrapolated and plotted considering the time complexity of the classical and the quantum algorithms as $O(n!)$ and $O(\text{poly}(n))$ respectively [12].

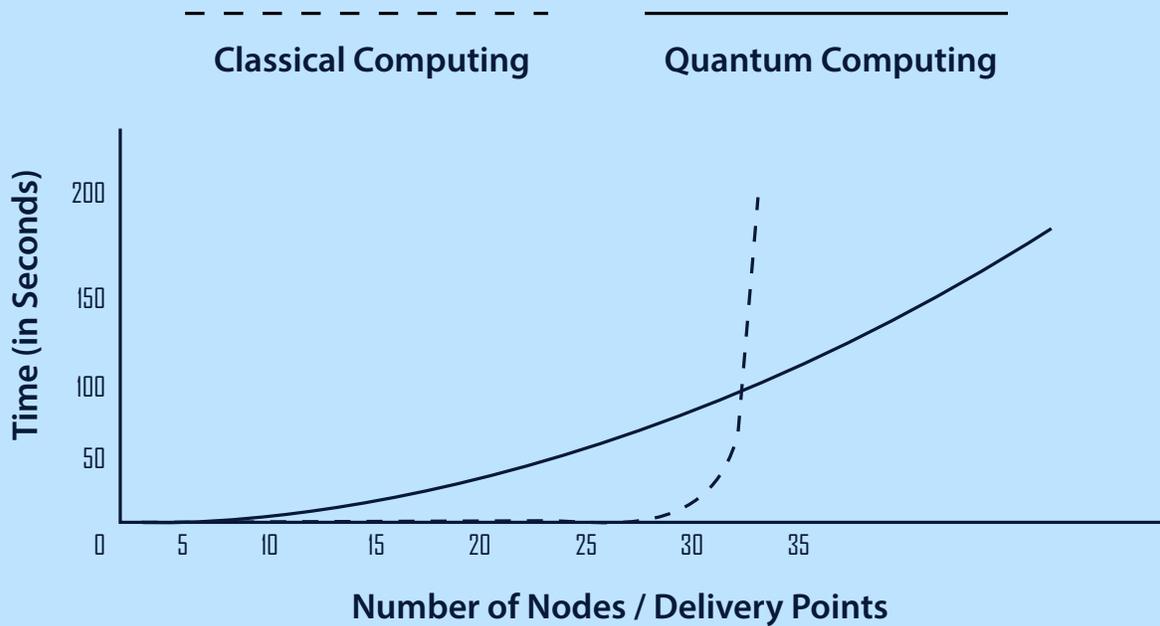


Figure 02: Time Complexity of Classical vs Quantum





Inference

The graph shows that for a VRP of up to 30 graphical nodes, the classical computer computes the solution in a practical time and faster than quantum. But post that, the computation time increases rapidly due to the exponential growth of computations with every increase of node or data point and the speed of classical computers drops drastically. After the 33rd graphical node or 1089 decision variables of the mathematical model, the quantum computer surpasses the speed of the classical computer and solves the problem faster. Since to solve the problem, the quantum computer takes polynomial time, it remains stable even with the increased number of data points or nodes and calculates the optimal solution in practical time.

Conclusion

The Vehicle Routing Problem has applications to various industries such as FMCG industries, Courier and Logistics, Fleet management industries, Disaster management supply chains, Freight Routing, Job scheduling, allocations, etc. These diverse applications of VRP in various industries make it a critical problem for businesses to solve. Currently, classical computers use approximations to arrive at a solution that is near minimum but not optimum. This problem can be addressed with the application of Quantum Computers.

Quantum computers are all set to achieve breakthrough advances in processing power and capabilities through the properties of quantum physics like

superposition and entanglement which is not possible by classical computation. This simulation empowers quantum machines to achieve acceleration in computing solutions for optimization problems such as VRP which involves evaluation of a large number of potential solutions and selecting the optimal ones. Efficient route optimization can help an organization speed up deliveries and cut costs by saving significantly on planning times, fuel costs, and driver wages. Even at a nascent stage, quantum computers are very robust and are destined to bring a major transformation in the logistics and supply chain industries with promising agility and efficiency.

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